BISHOP HARTMANN ACADEMY

SESSION - 2021 - 2021

CLASS - IX SUB - MATHEMATICS

CHAPTER - 1 → RATIONAL AND IRRATIONAL NUMBERS

* POINTS TO REMEMBER :

- A number which can be expressed as $\frac{a}{b}$, where 'a' and 'b' both are integers and 'b' is not equal to zero, is called a rational number.

$$\therefore Q = \left\{ \frac{a}{b} : a, b \in Z \text{ and } b \neq 0 \right\}$$

* There are infinitely many rational numbers between each pair of rational numbers.

* Method for finding large numbers of rational numbers between two given rational numbers.

(i) For any two rational numbers a and b, $\frac{a+b}{2}$ is also a rational number which lies between a and b. Thus
if $a > b \Rightarrow a > \frac{a+b}{2} > b$ and if $a < b \Rightarrow a < \frac{a+b}{2} < b$

(ii) For rational numbers $\frac{a}{b}$ and $\frac{c}{d}$, $\frac{a+c}{b+d}$ is also a rational number with its value lying between $\frac{a}{b}$ and $\frac{c}{d}$.

(iii) In ~~oder~~ order to find n rational numbers between x and y, first of all find $d = \frac{y-x}{n+1}$ If $x < y$. Then n rational numbers between ~~x~~

x and y are:

$x+d, x+2d, x+3d \cdots\cdots x+nd$. (2)

* If the denominator of a rational number can be expressed as $2^m \times 5^n$; where m and n both are whole numbers, the rational number is convertible into terminating decimal.

eg. $\frac{17}{50}$; since $50 = 2\times5\times5 = 2^1 \times 5^2$

i.e. 50 can be expressed as $2^m \times 5^n$

$\therefore$ Rational number $\frac{17}{50}$ is a terminating decimal.

* IRRATIONAL NUMBERS :

The square roots, cube roots, etc. of natural numbers are irrational numbers, if their exact values cannot be obtained.

$\sqrt{m}$ is irrational, if exact square root of m does not exist. similarly

$\sqrt[3]{m}$ is irrational, if exact cube root of m does not exist.

* SURDS (RADICALS)

If x is a positive rational number and n is a positive integer such that $x^{\frac{1}{n}}$ i.e. $\sqrt[n]{x}$ is irrational; then $x^{\frac{1}{n}}$ is called a surd.

e.g: $\sqrt[3]{6}$, $\sqrt[4]{8}$, $\sqrt{5}$

* RATIONALISATION :

when two surds are multiplied together such that their product is a rational number, the two surds are called rationalising factor of each other.

The process of rationalising a surd by multiplying it with its rationalising factor is called rationalisation.

(3)

e.g → $5\sqrt{2} \times 3\sqrt{2} = 15 \times 2 = 30$, which is rational number. ∴ $5\sqrt{2}$ and $3\sqrt{2}$ are rationalising factor of each other.

* SIMPLIFYING AN EXPRESSION BY RATIONALISING ITS DENOMINATOR :

- Multiply and divide the given expression by the least rationalising factor of its denominator. Simplify, if necessary.

e.g Rationalise the denominator of

(i) $\frac{5}{2\sqrt{2}}$ (ii) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

soln : →

(i) $\frac{5}{2\sqrt{2}}$, The least rationalising factor of $2\sqrt{2}$ is $\sqrt{2}$

$$\therefore \frac{5}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{2}}{2 \times 2} = \frac{5\sqrt{2}}{4} \text{ Ans}$$

(ii) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$, The least rationalising factor of $\sqrt{3}+\sqrt{2}$ is $\sqrt{3}-\sqrt{2}$

$$= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

$$= \frac{(\sqrt{3}-\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{(\sqrt{3})^2+(\sqrt{2})^2-2\sqrt{3}\times\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}$$

$$= \frac{3+2-2\sqrt{6}}{3-2}$$

$$= \frac{5-2\sqrt{6}}{1}$$

Ex 1-A

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Q.5: Without doing any actual division, find which of the following rational numbers have terminating decimal representation.

Solⁿ: Given number is $\frac{7}{16}$

since $16 = 2 \times 2 \times 2 \times 2 = 2^4 = 2^4 \times 5^0$

i.e. 16 can be expressed as $2^m \times 5^n$

$\therefore \frac{7}{16}$ is convertible into the terminating decimal.

Q.7. Given number is $\frac{32}{45}$

since $45 = 3 \times 3 \times 5 = 3^2 \times 5^1$

i.e. 45 can not be expressed as $2^m \times 5^n$

$\therefore \frac{32}{45}$ is not convertible into the terminating decimal.

Ex 1-B

Q.1 State, whether the following numbers are rational or not:

Solⁿ: (i) $(2+\sqrt{2})^2$

By using the formula $(a+b)^2 = a^2+b^2+2ab$

$(2+\sqrt{2})^2 = (2)^2 + (\sqrt{2})^2 + 2 \times 2 \times \sqrt{2}$

$= 4 + 2 + 4\sqrt{2}$

$= 6 + 4\sqrt{2}$ irrational

(vi) $\left(\frac{\sqrt{7}}{6\sqrt{2}}\right)^2 = \frac{7}{36 \times 2} = \frac{7}{72}$ Rational

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Q. 2 Find the square of

Solⁿ :- (i) $\left(\frac{3\sqrt{5}}{5}\right)^2 = \frac{(3)^2 \times (\sqrt{5})^2}{(5)^2}$

$= \frac{9 \times 5}{25}$

$= \frac{9}{5}$

$= 1\frac{4}{5}$

Q. 12 Write in ascending order :

Solⁿ: (ii) $2\sqrt[3]{5}$ and $3\sqrt[3]{2}$

$2\sqrt[3]{5} = \sqrt[3]{2^3 \times 5} = \sqrt[3]{8 \times 5} = \sqrt[3]{40}$

$3\sqrt[3]{2} = \sqrt[3]{3^3 \times 2} = \sqrt[3]{27 \times 2} = \sqrt[3]{54}$

$\because 40 < 54$ $\therefore \sqrt[3]{40} < \sqrt[3]{54}$

$\Rightarrow 2\sqrt[3]{5} < 3\sqrt[3]{2}$

Q. 15 COMPARE :

Solⁿ (ii) $\sqrt{24}$ and $\sqrt[3]{35}$

$\sqrt{24} = (24)^{\frac{1}{2}}$ and $\sqrt[3]{35} = (35)^{\frac{1}{3}}$

L.C.M of 2 and 3 is 6

$\frac{1}{2} \times \frac{3}{3} = \frac{3}{6}$, $\frac{1}{3} \times \frac{2}{2} = \frac{2}{6}$

$\Rightarrow (24)^{\frac{3}{6}} = (24^3)^{\frac{1}{6}} = (13824)^{\frac{1}{6}}$

$\Rightarrow (35)^{\frac{1}{3}} = (35)^{\frac{2}{6}} = (35^2)^{\frac{1}{6}} = (1225)^{\frac{1}{6}}$

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$\Rightarrow$ $13824 > 1225$

$\Rightarrow$ $\sqrt{24} > \sqrt[3]{35}$

Ex 1-C

Q.2 Write the lowest rationalising factor of

sol^n

(i) $5\sqrt{2}$

$= 5\sqrt{2} \times \sqrt{2} = 5 \times 2 = 10$

$\therefore$ lowest R.F $= \sqrt{2}$

(ii) $\sqrt{24} = \sqrt{2 \times 2 \times 2 \times 3}$

$= \sqrt{2^2 \times 6}$

$= 2\sqrt{6}$

$= 2\sqrt{6} \times \sqrt{6}$

$= 2 \times 6$

$= 12$

$\therefore$ lowest R.F $= \sqrt{6}$

(v) $\sqrt{18} - \sqrt{50} = \sqrt{3 \times 3 \times 2} - \sqrt{5 \times 5 \times 2}$

$= 3\sqrt{2} - 5\sqrt{2}$

$= -2\sqrt{2}$

$= -2\sqrt{2} \times \sqrt{2}$

$= -2 \times 2$

$= -4$

$\therefore$ lowest R.F $= \sqrt{2}$

(ix) $3\sqrt{2} + 2\sqrt{3}$

$= (3\sqrt{2} + 2\sqrt{3})(3\sqrt{2} - 2\sqrt{3})$

$= (3\sqrt{2})^2 - (2\sqrt{3})^2$

$= 9 \times 2 - 4 \times 3$

$= 18 - 12$

$= 6$ $\therefore$ lowest R.F $= (3\sqrt{2} - 2\sqrt{3})$

(3) Rationalise the denominators of :

sol^n (v) $\frac{2-\sqrt{3}}{2+\sqrt{3}}$

$= \frac{2-\sqrt{3}}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{(2-\sqrt{3})^2}{(2)^2-(\sqrt{3})^2}$

$= \frac{(2)^2+(\sqrt{3})^2 - 2\times2\times\sqrt{3}}{4-3}$

$= \frac{4+3-4\sqrt{3}}{1}$

$= 7-4\sqrt{3}$ Ans

(ix) $\frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}}$

$= \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}-3\sqrt{2}} \times \frac{2\sqrt{5}+3\sqrt{2}}{2\sqrt{5}+3\sqrt{2}}$

$= \frac{(2\sqrt{5}+3\sqrt{2})^2}{(2\sqrt{5})^2-(3\sqrt{2})^2}$

$= \frac{(2\sqrt{5})^2+(3\sqrt{2})^2+2\times2\sqrt{5}\times3\sqrt{2}}{4\times5 - 9\times2}$

$= \frac{20+18+12\sqrt{10}}{20-18}$

$= \frac{38+12\sqrt{10}}{2}$

$= \frac{\cancel{2}(19+6\sqrt{10})}{\cancel{2}}$

$= 19+6\sqrt{10}$ Ans

(4) Find the value of 'a' and 'b' in each of following :

(i) $\frac{2+\sqrt{3}}{2-\sqrt{3}} = a+b\sqrt{3}$

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$$\Rightarrow \frac{2+\sqrt{3}}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = a+b\sqrt{3}$$

$$\Rightarrow \frac{(2+\sqrt{3})^2}{(2)^2-(\sqrt{3})^2} = a+b\sqrt{3}$$

$$\Rightarrow \frac{(2)^2+(\sqrt{3})^2+2\times 2\times\sqrt{3}}{4-3} = a+b\sqrt{3}$$

$$\Rightarrow \frac{4+3+4\sqrt{3}}{1} = a+b\sqrt{3}$$

$$\Rightarrow 7+4\sqrt{3} = a+b\sqrt{3}$$

On comparing L.H.S and R.H.S we get

$a = 7$ $\quad$ $b\sqrt{3} = 4\sqrt{3}$

$$b = \frac{4\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = 4$$

$a = 7, \quad b = 4$

(5) simplify :-

$$\frac{22}{2\sqrt{3}+1} + \frac{17}{2\sqrt{3}-1}$$

By taking L.C.M

$$= \frac{22(2\sqrt{3}-1)+17(2\sqrt{3}+1)}{(2\sqrt{3}+1)(2\sqrt{3}-1)}$$

$$= \frac{44\sqrt{3}-22+34\sqrt{3}+17}{(2\sqrt{3})^2-(1)^2}$$

$$= \frac{44\sqrt{3}+34\sqrt{3}-22+17}{12-1}$$

$$= \frac{78\sqrt{3}-5}{11}$$ Ans

PORTION FOR I^{st} UNIT TEST - CH-1
RATIONAL AND IRRATIONAL NUMBER

ASSIGNMENT - ~~cop~~ complete the chapter