Class - XII   Subject - Math

FUNCTIONS :-

Functions are special case of relation.

Defn:- If X, Y are two non-empty sets then a subset f of $X \times Y$ is called a function from X to Y iff for each $x \in X$, there exist a unique $y \in Y$ such that $(x, y) \in f$.
We can write it as

$$f : X \rightarrow Y$$

Thus, a subset f of $X \times Y$ is called a function from X to Y iff

(i) for each $x \in X$, there exist $y \in Y$ such that $(x, y) \in f$

(ii) no two different ordered pairs have the same first component.

(2)

* Image of an element :-

The unique element $y \in Y$ is called the image of the element x of X under the function $f : X \to Y$. It is denoted by $f(x)$

i.e $y = f(x)$

* Domain & Range of a function :-

Let f be a function from X to Y, then the set X is called the domain of the function f and the set Y is called the co-domain of the function.

The set consisting of all the images of the elements of X under the function f is called the range of the function.

i.e range of $f = \{ f(x) : \forall x \in X \}$

Notes :- If a & b are real numbers such that $a < b$, then

(i) $(x-a)(x-b) < 0$ iff $a < x < b$ i.e $x \in (a,b)$

(ii) $(x-a)(x-b) \leq 0$ iff $a \leq x \leq b$ i.e $x \in [a,b]$

(iii) $(x-a)(x-b) > 0$ iff $x \in (-\infty, a) \cup (b, \infty)$

(iv) $(x-a)(x-b) \geq 0$ iff $x \in (-\infty, a] \cup [b, \infty)$

(3)

Q 1> find the domains of the functions :-

$$f(x) = \frac{x^2+2x+1}{x^2-8x+12}$$

Soln:- $f(x) : \frac{x^2+2x+1}{x^2-8x+12}$

For D_f, $f(x)$ must be real no.

$\Rightarrow \frac{x^2+2x+1}{x^2-8x+12}$ must be a real no.

$\Rightarrow x^2-8x+12 \neq 0$

$\Rightarrow x^2-6x-2x+12 \neq 0$

$\Rightarrow (x-2)(x-6) \neq 0$

$\Rightarrow x \neq 2, 6$

$\Rightarrow D_f = R - \{2, 6\}$ Ans

Q 2> find the range of the function $f(x) = 2-3x$, $x \in R$, $x > 0$

Soln:- $f(x) = 2-3x$, $x \in R$, $x > 0$

For R_f, let $y = f(x) = 2-3x$, $x \in R$, $x > 0$

(4)

Now, $x > 0$

$\Rightarrow 3x > 0$

$\Rightarrow -3x < 0$

$\Rightarrow 2 - 3x < 2$

$\Rightarrow y < 2$

$\Rightarrow R_f = (-\infty, 2)$

Ans

Q.3> find the domain and the range of the function $f(x) = \dfrac{x^2}{1+x^2}$.

Soln: $f(x) = \dfrac{x^2}{1+x^2}$

for D_f, $f(x)$ must be real number

$\Rightarrow \dfrac{x^2}{1+x^2}$ must be real no.

Now, for any value of x it always give real no.

$\therefore D_f = R$ [Real no.] Ans

for R_f, let $y = f(x)$

$\Rightarrow y = \dfrac{x^2}{1+x^2}$

$\Rightarrow x^2 y + y = x^2$

$\Rightarrow x^2 y - x^2 = -y$

$\Rightarrow x^2 (y-1) = -y$

$\Rightarrow x^2 = \dfrac{-y}{y-1}, \quad y \neq 1$

But $x^2 \geq 0$ for all $x \in R$

$\therefore \dfrac{-y}{y-1} \geq 0, \quad y \neq 1$

$\Rightarrow \dfrac{-y}{(y-1)} (y-1)^2 \geq 0 \, (y-1)^2$

Multiply both side by $(y-1)^2$

$\Rightarrow -y(y-1) \geq 0$

$\Rightarrow y(y-1) \leq 0$

$\Rightarrow (y-0)(y-1) \leq 0$

$\Rightarrow \cancel{\leq 0} \quad 0 \leq y \leq 1$ but $y \neq 0$

$\therefore 0 \leq y < 1$

$\Rightarrow R_f = [0, 1)$ Ans

Note:- Solve questions related to this from any book.

S	M	T	W	T	F	S
31					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30

Class - XII Sub. - Math

* Types of functions :-

① One-one function - A function 'f' from X to Y is called one-one (or injective) iff different elements of X have different images in Y.

i.e iff $x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$ for all $x_1, x_2 \in X$

Or, iff $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$ for all $x_1, x_2 \in X$

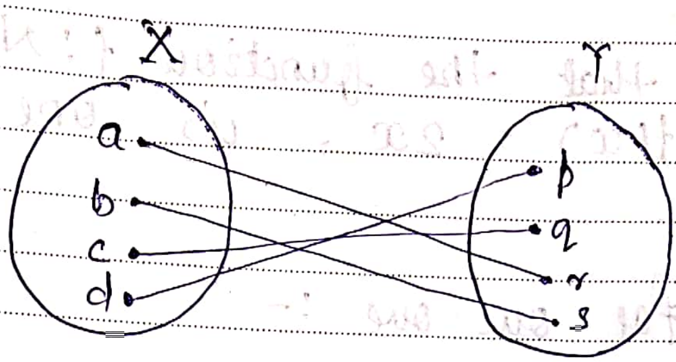
② Onto function - A function 'f' from X to Y is called onto (or surjective) iff each element of Y is the image of atleast one element of X.

i.e iff codomain of f = range of f

i.e $Y = f(X)$

③ One-one Onto function :- If function is both one-one & onto then it is bijective function.

Ex:-



It is one-one function because different elements of X have different images in Y.

It is onto function because each element of Y is the image of an element X

or, range of f = {p, q, r, s} = co domain of f.

* Even and odd functions :-

A real function f is called a even function iff $f(-x) = f(x)$ for all $x \in D_f$ and f is called an odd function iff $f(-x) = -f(x)$ for all $x \in D_f$

(8)

Q.1> Show that the function $f: N \rightarrow N$, defined by $f(x) = 2x$, is one-one but not onto.

Soln:- For one-one :-

Let $x_1, x_2 \in N$ such that

$$f(x_1) = f(x_2)$$

$$\Rightarrow 2x_1 = 2x_2$$

$$\Rightarrow x_1 = x_2$$

$\Rightarrow$ f is one-one function.

For onto :-

f is not onto because as $1 \in N$ (codomain of f) and there does not exist any $x \in N$ (domain of f) such that $f(x) = 1$, so it is not onto.

Note:- Solve the questions based on it.